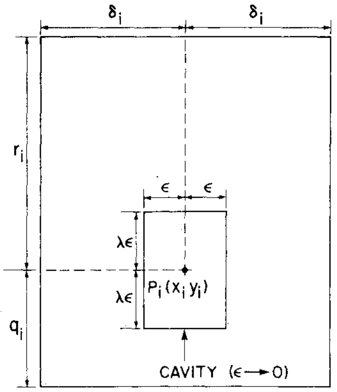

Fig. 1 Location of mesh point in rectangular element.

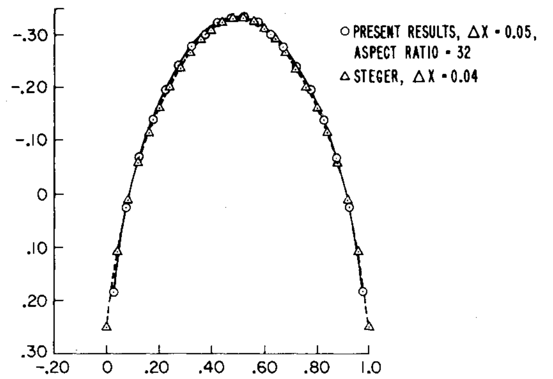

Fig. 2 Coefficient of pressure for a parabolic-arc airfoil of thickness ratio 0.06, at $M_\infty = 0.825$.

integration of Eq. (3). In most cases, however, the integral has to be evaluated numerically. Radbill[4] describes a numerical integration scheme suitable for an arbitrary sharp-edged airfoil.

The ideas described in this article were implemented for a nonlifting parabolic-arc airfoil of thickness ratio $\bar{\delta} = 0.06$. Discussion of the results is best done through aspect ratios of the elements (Fig. 1).

$$\text{aspect ratio of } \Delta_j = (\text{height of } \Delta_j)/(\text{width of } \Delta_j)$$

$$= (r_j + q_j)/2\delta_j \tag{23}$$

Figure 2 shows the coefficient of pressure plots at freestream Mach number 0.825, for the present method, using 280 high-aspect ratio elements, and for a finite-difference method by J. L. Steger of NASA Ames Research Center, using a network of 150×64 mesh points. In the transformed coordinates, the region of integration extended about one chord length from the airfoil edges, in the streamwise direction, and eight chord lengths in the transverse direction.

Convergence was attained in 11 iterations for $M_\infty = 0.825$ and in less than 12 iterations for all subcritical Mach numbers tested. Most of the computing time was used in fulfilling the convergence criterion for mesh points near the airfoil. Consequently, the time per iteration was substantially decreased by not computing $g[u^{(m)}]$ in Eq. (17b) for any mesh point where the convergence criterion was already satisfied. The effect on the coefficient of pressure was insignificant.

For supercritical flows up to $M_\infty = 0.87$, symmetrical results were yielded. Extension of the method to calculation of supercritical flows with shocks has been made very recently and will be reported in a future publication.

Acknowledgment

J.R. Spreiter of Stanford University is gratefully acknowledged for his advice when this work was being researched.

References

[1] Ogana, W., "Computation of Steady Two-Dimensional Transonic Flows by an Integral Equation Method," Ph.D dissertation, 1976, Stanford Univ., Stanford, Calif.

[2] Spreiter, J.R. and Alksne, A.Y., "Theoretical Predictions of Pressure Distributions on Nonlifting Airfoils at High Subsonic Speeds," NACA Rept. 1217, 1955.

[3] Nixon, D. and Hancock, G.J., "High Subsonic Flow Past a Steady Two-Dimensional Aerofoil," ARC CP 1280, 1974.

[4] Radbill, J.R., "Solution of Subsonic Transonic Wing Flow by an Integral Equation Method," Space Division, North American Rockwell Corp., Downey, Calif., SD70-121, 1970.

[5] Norstrud, H., "High-Speed Flow Past Wings," NASA CR-2246, 1973.

[6] Nixon, D., "Extended Integral Equation Method for Transonic Flows," *AIAA Journal*, Vol. 13, July 1975, pp.934-935.

[7] Nixon, D., "Calculation of Transonic Flows Using an Extended Integral Equation Method," AIAA Paper 75-876, Hartford, Conn., June 1975; this issue, pp. 934-935.

[8] Lynn, M.S. and Timlake, W.P., "The Numerical Solution of Singular Integral Equations of Potential Theory," *Numerische Mathematik*, Vol. 11, 1968, pp. 77-98.

[9] Isaacson, E. and Keller, H.B., Ch. 3, *Analysis of Numerical Methods*, Wiley, New York, 1966.

Technical Comments

Comment on "Flutter of a Panel Supported on an Elastic Foundation"

Alexander H. Flax*
Institute for Defense Analyses, Arlington, Va.

CHOPRA[1] has given relationships between theoretical frequency-damping coefficient values on the flutter boundary for a panel supported on an elastic foundation in terms of the values for the same panel unsupported. For supersonic panel flutter, when aerodynamic forces based on linearized quasistatic or piston theory are assumed, such relationships are inherent in the parameters of the theoretical analysis of the flutter problem as given by numerous authors, including Movchan[2] and Dugundji[3] (Ref. 6 of Chopra's Note). Unfortunately, Chopra's statement of the relationships is in error and he has also apparently assumed that they apply to more general theories for the aerodynamic forces and to subsonic panel flutter, which is not the case.

On the flutter boundary, where the variation of the motion in time is purely sinusoidal, the appropriate relationships between the flutter frequencies ω_F and damping coefficients g_T (which, as defined by Dugundji, is a viscous damping coefficient and not the so-called structural damping coefficient usually denoted by g) are, in the notation used by Dugundji and Chopra

$$\omega_{Fe}^2 = \omega_F^2 + K/m \tag{1}$$

and

$$\omega_{Fe} g_{Te} = \omega_F g_T \tag{2}$$

where K is the stiffness of the foundation per unit area of panel, m is the mass per unit area of the panel, and the subscript e refers to the panel with elastic support.

Received Jan. 5, 1976.
Index category: Panel Flutter.
*President. Fellow AIAA.

The damping coefficient g_T may be composed in general of aerodynamic, structural, and even conceivably mechanical or electrical damping, and is closely related to the viscous damping coefficient of classical mechanics, B, defined[2] such that the transverse damping force per unit area of panel is given by $B\mathrm{d}w/\mathrm{d}t$, where $\mathrm{d}w/\mathrm{d}t$ is the transverse velocity of the panel. Dugundji's G_s is the structural part of B and, in general

$$g_T\omega_0 m = B$$

where ω_0 is a reference frequency. It may be noted that g_T is actually an arbitrary number depending on the reference frequency, ω_0, which is chosen.

These relationships hold when all other parameters of the panel flutter problem, including the velocity parameter, are held constant. Thus the flutter velocity is held constant, while the frequency is increased and the damping coefficient is decreased in accordance with Eqs. (1) and (2) when the panel is subjected to elastic support. The damping coefficient of the supported panel is given explicity by

$$g_{Te} = (\omega_F/\omega_{Fe})g_T$$

or

$$g_{Te} = g_T/\left(1+\frac{K}{m\omega_F^2}\right)^{1/2} \tag{3}$$

This last equation differs from the one given by Chopra in that the term in parenthesis is in the denominator rather than the numerator, thereby leading to a lower damping coefficient with elastic support than without. Figure 8 of Dugundji's paper[3] illustrates this relationship. It is important to point out that in application of these relationships, care must be taken to insure that the eigenvalue branch, or mode, leading to the lowest damping coefficient at the flutter point is considered.

Chopra's interpretation of the increase in damping coefficient with increasing elastic support was that it reflected an increase in total damping of the panel introduced by the elastic foundation, leading in turn to an increase in flutter velocity. The proper interpretation of g_{Te}, however, is that it represents the value of damping coefficient such that, with all other parameters of the panel unchanged, including the velocity parameter, but with the addition of the elastic support represented by K, the panel is left at a point on the flutter boundary. Moreover, the deflection, w, of the panel is given by[2,3]

$$L(w) - \lambda(\partial w/\partial\xi) + \Omega^2 w = 0 \tag{4}$$

where $L(w)$ is the structural (self-adjoint) partial differential operator of the panel, λ is the velocity parameter, ξ is the dimensionless variable in the direction of the air velocity and Ω^2 is the eigenvalue at the flutter boundary, which is, in general, complex. The effect of Eqs. (1) and (2) in going from the unsupported to the elastically supported panel is to leave Ω^2 unchanged. Thus, the complex eigenfunction, w, is also unchanged, and the extend to which the motion tends twoard either static oscillation or the travelling wave type of motion is determined by the degree of variation in the phase of w along the panel. This phase variation is determined by Eq. (4) and is not altered by the transformation represented in Eqs. (1) and (2), contrary to the suggestion in Chopra's Note.

Finally, no distinction is made in Ref. 1 between supersonic panel flutter with quasistatic aerodynamic forces which is discussed above and subsonic panel flutter. The citation of the paper by Dugundji, Dowell, and Perkin[4] and other analyses of subsonic panel flutter by Chopra leaves the impression that the relationships appropriate to the special supersonic case discussed above may also apply to subsonic panel flutter. This, however, is not in general true. For the relatively simple subsonic aerodynamic forces for a two-dimensional panel used by Dugundji, Dowell, and Perkin,[4] it is clear from a dimensional analysis that introduction of elastic support characterized by a spring constant, K, requires not only the relationship of Eq. (1) but also

$$\frac{B_e\omega_{Fe}}{m_e} = \frac{B\omega_F}{m} \tag{5}$$

$$\frac{u_F}{\omega_F\ell} = \frac{u_{Fe}}{\omega_{Fe}\ell} \tag{6}$$

and

$$\frac{\rho_e\ell}{m_e(1-K/m_e\omega_{Fe}^2)} = \frac{\rho\ell}{m} \tag{7}$$

where u is the flutter velocity, ρ is the air mass density, and ℓ is a characteristic length. Thus, the flutter velocity of the supported panel in the subsonic case will increase in the same proportion as the flutter frequency by virtue of Eq. (6) and the flutter point will correspond to a corrected value of air mass density ρ_e in accordance with Eq. (7). In the special cases of supersonic flutter discussed above for which Eqs. (1-3) constitute a valid set of transformations, the aerodynamic forces are proportional to the product ρu and neither ρ nor u appear spearately in the equations.

As is shown in Refs. 4 and 5, for panels in incompressible flow, static divergence does not depend on mass and corresponds to $\omega_F = 0$. Thus the relationships of Eqs. (1, 5, 6, and 7) cannot be used to predict the change in panel divergence speed brought about by the addition of an elastic foundation. Less general corrections can be found for specific cases. For example, the simply supported two-dimensional panel treated in Refs. 4 and 5 undergoes static divergence when

$$D\frac{n^4\pi^4}{\ell^4} + T\frac{n^2\pi^2}{\ell^2} + K - \frac{n\pi}{\ell}\rho u_0^2 = 0$$

where D is the plate bending stiffness, T is the plate tension in the flow direction, ℓ is the length between bays in the flow direction, n is the (integral) number of half waves in the buckling deflection. Clearly when $K=0$, the panel divergence velocity occurs for $n=0$. However, when K is large compared to D/ℓ^4 and T/ℓ^2 the lowest divergence velocity may occur for higher values of n. (This is similar to what occurs in the buckling of a column on an elastic foundation.) Thus, the deflection modes involved in panel static divergence and the subsequent panel flutter change as K increases to large values and the results of transformations such as discussed above are only representative of the lowest flutter speeds when the critical subsonic panel divergence and flutter modes do not shift with increasing K.

There is yet another cautionary note which must be sounded concerning the use of the relationships discussed above in the case of subsonic panel flutter. It is well known (e.g., Ref. 4) that in this case the effect of dissipative forces may be destabilizing. Thus, it may not always be apparent on which side of a stability boundary higher or lower damping coefficients may lie. Although this difficulty does not, of itself, change the significance of the stability boundary, it does need to be considered carefully in application of stability boundaries to determine the stability of given structures with given properties, including damping coefficients.

References

[1] Chopra, I., "Flutter of a Panel Supported on an Elastic Foundation," *AIAA Journal*, Vol. 13, May 1975, pp. 687-688.

[2] Movchan, A.A., "On the Stability of a Panel Moving in a Gas," translated and issued as NASA Republication RE11-21-58W, 1958.

[3] Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, July 1966, pp. 1257-1266.

[4]Dugundji, J., Dowell, E., and Perkin, B., "Subsonic Flutter of Panels on Continuous Elastic Foundation," *AIAA Journal*, Vol. 1, May 1963, pp. 1146-1154.
[5]Flax, A.H., "Aero and Hydro-Elasticity," *Structural Mechanics, Proceedings of the First Symposium on Naval Structural Mechanics*, Pergamon Press, New York, 1960.

Reply by Author to A.H. Flax

Inderjit Chopra*
Massachusetts Institute of Technology, Cambridge, Mass.

THE author appreciates the interest of A.H. Flax in this work. The analysis made in Ref. 1 is strictly true only for a panel exposed to supersonic flow on one of its surfaces. In the Note, g_T is the total damping coefficient and it is a combination of structural damping and aerodynamic damping coefficients, as defined in Ref. 2. In the author's paper, it has been shown that the addition of linear springs support to the panel can be interpreted as if only g_T of the basic panel is changed. Thus, if one knows the flutter velocity parameter λ variation with g_T for a panel with no elastic support, then it is possible to get a new λ for a panel resting on linear elastic foundation from the results of basic panel without doing any extra calculations.

Received July 29, 1976.
Index category: Panel Flutter.
*Research Assistant, Department of Aeronautics and Astronautics.

Flax has given an alternate way of showing the effect of elastic foundation on the flutter of a panel by changing g_T such that same λ will result. Either way can be used and appears equally valid.

References

[1]Chopra, I., "Flutter of a Panel Supported on an Elastic Foundation," *AIAA Journal*, Vol. 13, May 1975, pp. 687-688.
[2]Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, July 1966, pp. 1257-1266.

Errata

Perturbation Method for Transonic Flows about Oscillating Airfoils

R. M. Traci, E. D. Albano, J. L. Farr Jr.
Science Applications, Inc., El Segundo, Calif.
[AIAA J., **14**, 1258-1265, (1976)]

IN the subject paper, reference is made to "bold-face type" and "underlined terms." Regrettably, such typography did not appear. The terms referred to are all those where "***k***" appears in Eqs. (3-5, 10, and 14).

Received Oct. 21, 1976.
Index categories: Nonsteady Aerodynamics; Subsonic and Transonic Flow.